Self-affine case for the roughness effect on the frictional force in boundary lubrication

George Palasantzas
Physics Department, Northeastern University, Boston, Massachusetts 02115
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We comment on the analytic expressions obtained by Daikhin and Urbakh [Phys. Rev. E **49**, 1424 (1994)] for the case where the wall roughness is described by self-affine structure for the frictional forces in confining geometry systems.

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Very recently Daikhin and Urbakh [1] have shown that the presence of roughness essentially *increases* the frictional force in confining systems, and also can lead to the time dependence of the friction. We investigate here the static part of the frictional force between two walls separated by a thin liquid film in the case where the wall roughness is characterized by a self-affine structure. Furthermore, we shall try to derive the corresponding analytical expressions. Daikhin and Urbakh [1] considered the case of walls bounded by rough surfaces with a Gaussian height-height correlation $C(R)=\sigma^2 e^{-R^2/\xi^2}$, which in the case of weak roughness $\sigma \ll \xi$ and wall separation $d \ll \xi$ yielded analytical expressions. The parameters σ and ξ are respectively the normal rms roughness and the in-plane correlation length.

However, a wide variety of surfaces and interfaces occurring in nature are well represented by a kind of roughness associated with self-affine fractal scaling [2]. If a surface exhibits self-affine roughness [3], the height difference correlation function $g(R)$ will scale as $g(R)\propto R^{2H}$ if $R \ll \xi$, and $g(R)\approx 2\sigma^2$ if $R \gg \xi$. The function $g(R)$ is related to the height-height correlation $C(R)$ by $g(R)=2\sigma^2-2C(R)$. The exponent $0<H<1$ is referred to as the "roughness" exponent [4,5]. Small values of $H\sim 0$ represent extremely jagged or irregular surfaces, while large values of $H\sim 1$ represent smooth hill-valley structures. In fact, the Gaussian correlation can be considered as a limiting case of the fractal correlation $C(R)\sim e^{-(R/\xi)^{2H}}$ for $H=1$. In our calculations for self-affine fractals, we shall consider the *k-correlation* model that in *k*-space has the form

$$C(\mathbf{k})=(2\pi)\sigma^2\xi^2/(1+ak^2\xi^2)^{1+H}$$

with $a=(1/2H)[1-(1+ak_c^2\xi^2)^{-H}]$ for $H>0$, and $a=1/2\ln(1+ak_c^2\xi^2)$ for $H=0$, as well as $k_c=\pi/a_0$, with a_0 the atomic spacing [5].

Let us denote the velocity of motion of the upper wall by V_0 and the liquid viscosity by n. The static part of the frictional force is given by [1]

$$F_{st}=(nV_0/d)\left[1+(2\pi)^{-2}d^{-2}\sum_{i=1}^{2}\int C_i(\mathbf{k})a(kd,\phi)d^2\mathbf{k}\right], \quad (1)$$

with $\int_0^{2\pi}a(kd,\phi)d\phi\approx\frac{5}{2}(2\pi)-7(kd)^2/10$ for $d\ll\xi_i$. The indices $i=1,2$ refer to the upper and lower walls, and nV_0/d represents the frictional force for flat walls. The fulfillment of the condition $d\ll\xi$ is not sufficient for the case of power law roughness to ensure experimentally realizable values for $d\sim 5$ nm in order that $F_{st}\geq nV_0/d$. In fact, depending on the value of H the perturbation expansion of $a(kd,\phi)$ has to continue to higher order terms. For self-affine fractals F_{st} is given by

$$F_{st}=\frac{nV_0}{d}\left(1+\frac{1}{d^2}F_1-F_2\right), \quad (2)$$

$$F_1=\tfrac{5}{2}\sum_{i=1}^{2}\sigma_i^2,$$

$$F_2=\tfrac{7}{10}\sum_{i=1}^{2}\frac{\sigma_i^2}{2a_i\xi_i^2}\times\left\{\frac{1}{a_i(1-H_i)}[(1+a_ik_{ci}^2\xi_i^2)^{1-H_i}-1]-2\right\}, \quad (3)$$

where its applicability up to second order in $a(kd,\phi)$ is limited to wall separations $d\ll d_c=(F_1/F_2)^{1/2}$($F_1 \gg d^2F_2$). In order to gain a feeling of the effect, of H, as well as the applicability of the previous results to nanoscale roughness, we present some estimations of d_c. In the following, we shall consider $H_{1,2}=H$ for illustration purposes. For $\xi_{1,2}=1000$ nm and $\sigma_{1,2}=100$ nm, $d_c\geq 36$ nm for $H\geq 0.6$. For $\xi_{1,2}=100$ nm and $\sigma_{1,2}=10$ nm, $d_c\geq 9$ nm for $H\geq 0.6$. Therefore, the previous estimations show that for $H<0.5$ higher order terms [or the exact expressions for $a(kd,\phi)$] are required in order for experimentally reasonable values of $d\ll d_c$ to be feasible ($d\sim 5$ nm [1,6] for wall roughness on the nanoscale range.

The dependence of the frictional force on the particular choice of the correlation function is given by its Fourier $C_i(k)$ in Eq. (1). (For various forms of the correlation function see Refs. [5] and [7].) From Eq. (3), the term F_2 includes, besides the ratio σ_i^2/ξ_i^2, the roughness exponent H. If we consider $H=0.5$, for $a_0\ll\xi$ the analytic *k-correlation* model used in this study yields the simple correlation function $C(R)=\sigma^2e^{-R/\xi}$ [5]. For $\sigma_{1,2}=10$ nm, $\xi_{1,2}=200$ nm, and $d=5$ nm, we obtain, for $H=0.5$ $F_1/d^2=20.0$ and $F_2=7.32$, as well as for

$H=0.8$ $F_1/d^2=20.0$ and $F_2=0.4$. Therefore, for a fixed ratio $\sigma/\xi \ll 1$ the larger the H the smaller is F_2, and as a result the dominant contribution to the frictional force comes from σ. However, for small values of H the term F_2 increases, resulting in higher sensitivity of the frictional force to the roughness exponent H.

In conclusion, our simple calculations show that in order to account for nanoscale roughness in confining geometry systems, the effect of possible surface fractality has to be taken into account in the perturbation expansions. Furthermore, we gave the expressions for this effect in terms of the theory developed by Daikhin and Urbakh [1].

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